

What is claimed is:

1: A Fourier transform processor responsive to N input data points to compute N data output points, said Fourier transform processor comprising:

a plurality of r parallel processing elements, each of said plurality of r parallel processing elements having N/r input data points and N/r output data points, said N/r input data points of said plurality of r parallel processors being coupled to respective ones of said N input data points, each of said plurality of r parallel processing elements being independent of each other and having at least two stages of butterfly computing elements; and

a combination phase computing element having N output points forming said N data output data points of said Fourier transform processor, said combination phase computing element having N input data points coupled to respective ones of said N/r output data points from said plurality of r processing elements, said combination phase computing element including a single stage of butterfly computing elements.

2. A Fourier transform processor in accordance with claim 1, wherein said single stage of butterfly computing elements in said combination phase computing element is represented by:

$$X_{l(k)} = \sum_{j_1=0}^{r-1} X_{pj_1(k)} w_N^{((j_1 N/r + j_1 k))_N} \quad (37)$$

for $l = 0, 1, \dots, r-1$, $k = 0, 1, \dots, \beta-1$, where $((x))_N$ denotes x modulo N

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a-1 3. A Fourier transform processor in accordance with claim 1, wherein each of said r parallel processing elements includes a butterfly processing element for use in a decimation in frequency radix- r butterfly responsive to a plurality of input data points, said radix- r butterfly providing a plurality of output data points, said butterfly processing element comprising:

a plurality of multipliers, each of said multipliers having a respective first and second input terminals and a respective output terminal; and

an adder having a plurality of input terminals and an output terminal, each of said plurality of input terminals of said adder being coupled to a respective one of said output terminals of said plurality of multipliers;

said plurality of first input terminals of said plurality of multipliers each being respectively coupled to one of said plurality of input data points, said output terminal of said adder being coupled to one of said plurality of output data points

said plurality of second input terminals of said plurality of multipliers being coupled to a set of coefficients derived from the product of an adder matrix T_r and a twiddle factor matrix W_N^r .

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Sub 4. A butterfly processing element in accordance with claim 3, wherein said coefficients derived from the product of said adder matrix T_r and said twiddle factor matrix W_N^r , are substantially given by

$$T_r = \begin{bmatrix} w^0 & w^0 & w^0 & - & w^0 \\ w^0 & w^{N/r} & w^{2N/r} & - & w^{(r-1)N/r} \\ w^0 & w^{2N/r} & w^{4N/r} & - & w^{2(r-1)N/r} \\ - & - & - & - & - \\ w^0 & w^{(r-1)N/r} & - & - & w^{(r-1)^2 N/r} \end{bmatrix} = [T_{(l,m)}] \quad (5A),$$

$$\text{where } T_{(l,m)} = w^{\langle (lm \frac{N}{r}) \rangle_N} \quad (6A),$$

and defining $W_{(r,k,i)}$ the set of the twiddle factor matrices W_N^r as:

$$W_{(r,k,i)} = \begin{bmatrix} w_{(0,k,i)} & 0 & - & 0 \\ 0 & w_{(1,k,i)} & - & 0 \\ - & - & - & - \\ 0 & 0 & - & w_{((r-1),k,i)} \end{bmatrix} = [w_{(l,m) \langle k,i \rangle}] \quad (8A),$$

in which,

$$w_{(l,m) \langle k,i \rangle} = w^{\langle (\tilde{N}(\frac{k}{r}) l r^i) \rangle_N} \quad \text{for } l = m, \text{ and } 0 \text{ elsewhere} \quad (9A),$$

wherein,

$$B_{r \text{ DIF}} = W_{(r,k,i)} \times T_r = [B_{r \text{ DIF}}(l,m) \langle k,i \rangle] \quad (10A),$$

$$\text{with } B_{r \text{ DIF}}(l,m) \langle k,i \rangle = w^{\langle (l m N/r + \tilde{N} (kr^i) l r^i) \rangle_N} \quad (11A),$$

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~~*air*~~ $l = m = 0, \dots, r-1, i = 0, 1, \dots, n-1, k = 0, 1, \dots, (N/r) - 1, ((x))_N$ denotes x modulo N and $\tilde{N}(k/r^i)$ is defined as the integer part of the division of k and r^i , whereby the operation of said butterfly processing element for said decimation in frequency radix- r butterfly is:

$$\text{the colon vector } X_{(r,k,i)} = B_{r \text{ DIF}} \times x = \begin{bmatrix} X_{(l)_{(k,i)}} \end{bmatrix} \quad (12A),$$

$$\text{where the } l^{\text{th}} \text{ output } X_{(l)_{(k,i)}} = \sum_{m=0}^{r-1} x_{(m)} w^{((l m N/r + \tilde{N}(k r^i) l r^i))_N} \quad (13A).$$

5. A Fourier transform processor in accordance with claim 1, wherein each of said r parallel processing elements includes a butterfly processing element for use in a decimation in time radix- r butterfly responsive to a plurality of r input data points, said radix- r butterfly providing a plurality of output data points, said butterfly processing element comprising:

a plurality of $r-1$ multipliers, each of said $r-1$ multipliers having a respective first and second input terminals and a respective output terminal; and

an adder having a plurality of r input terminals and an output terminal, one of said plurality of r input terminals of said adder being coupled to a first input data point of said plurality of r input data points, each of the $r-1$ others of said plurality of r input terminals of said adder being coupled to a respective one of said output terminals of said plurality of $r-1$ multipliers;

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said r-1 plurality of first input terminals of said r-1 plurality of multipliers each being respectively coupled to one of r-1 of said plurality of r input data points, said output terminal of said adder being coupled to one of said plurality of output data points; and

said plurality of second input terminals of said r-1 plurality of multipliers being coupled to a set of coefficients derived from the product of an adder matrix T_r and a twiddle factor matrix W_N^r .

6. An apparatus in accordance with claim 5, wherein said coefficients derived from the product of said adder matrix T_r and said twiddle factor matrix W_N^r , are substantially given by,

$$B_{r \text{ DIT}} = T_r \times W_{(r,k,i)} = \left[B_{r \text{ DIT}}(l,m)_{(k,i)} \right] \quad (14A),$$

$$\text{in which } B_{r \text{ DIT}}(l,m)_{(k,i)} = w_{((l \cdot m \cdot N/r + \bar{N} \cdot (k/r^{(n-1)}))_{m \cdot r^{(n-1)}}))_N} \quad (15A),$$

$$\text{and } W_{(r,k,i)} = \begin{bmatrix} w_{(0,k,i)} & 0 & - & 0 \\ 0 & w_{(1,k,i)} & - & 0 \\ - & - & - & - \\ 0 & 0 & - & w_{((r-1),k,i)} \end{bmatrix} = \left[w_{(l,m)_{(k,i)}} \right] \quad (16A),$$

where $w_{(l,m)_{(k,i)}} = w_{((\bar{N} \cdot (k/r^{(n-1)}))_{m \cdot r^{(n-1)}}))_N}$ for $l = m$, and 0 elsewhere (17A),

and $n = (\log N / \log r) - 1$, whereby the operation of said butterfly processing element for said decimation in time radix - r butterfly is:

$$\text{the colon vector } X_{(r,k,i)} = B_{r \text{ DIT}} \times x = \left[X_{(l)_{(k,i)}} \right] \quad (18A),$$

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where the l^{th} output $X_{(l)_{(k,i)}}$ = $\sum_{m=0}^{r-1} X_{(m)} w^{((l m N/r + \tilde{N} (k/r^{(n-i)}))_m r^{(n-i)})_N}$ (19A).

7. A Fourier transform processor in accordance with claim 1, wherein each of said r parallel processing elements includes a radix- r butterfly responsive to a plurality of input data points and having a plurality of calculation phases, said radix- r butterfly further providing a plurality of output data points, said radix- r butterfly comprising:

a first plurality of multipliers; and

a second plurality of adders, wherein

said first plurality of multipliers operate substantially simultaneously during a given calculation phase of said plurality of calculation phases, and said second plurality of adders operate during said plurality of calculation phases other than said given calculation phase.

8. An apparatus in accordance with claim 7 wherein said given calculation phase is the first calculation phase.

9. An apparatus in accordance with claim 7, wherein said plurality of multipliers are coupled to a set of coefficients derived from the product of an adder matrix T_r and a twiddle factor matrix w_N^r .

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10. An apparatus in accordance with claim 9, wherein said coefficients derived from the product of said adder matrix T_r and said twiddle factor matrix W_N^r , are substantially given by

$$T_r = \begin{bmatrix} w^0 & w^0 & w^0 & - & w^0 \\ w^0 & w^{N/r} & w^{2N/r} & - & w^{(r-1)N/r} \\ w^0 & w^{2N/r} & w^{4N/r} & - & w^{2(r-1)N/r} \\ - & - & - & - & - \\ w^0 & w^{(r-1)N/r} & - & - & w^{(r-1)^2 N/r} \end{bmatrix} = [T_{(l,m)}] \quad (5A),$$

$$\text{where } T_{(l,m)} = w^{((lm \frac{N}{r}))_N} \quad (6A),$$

and defining $W_{(r,k,i)}$ the set of the twiddle factor matrices W_N^r as:

$$W_{(r,k,i)} = \begin{bmatrix} w_{(0,k,i)} & 0 & - & 0 \\ 0 & w_{(1,k,i)} & - & 0 \\ - & - & - & - \\ 0 & 0 & - & w_{((r-1),k,i)} \end{bmatrix} = [w_{(l,m)}]_{(k,i)} \quad (8A),$$

in which,

$$w_{(l,m)(k,i)} = w^{((\tilde{N}(\frac{k}{r^i})l r^i))_N} \quad \text{for } l = m, \text{ and } 0 \text{ elsewhere} \quad (9A),$$

wherein,

$$B_{r \text{ DIF}} = W_{(r,k,i)} \times T_r = [B_{r \text{ DIF}}(l,m)]_{(k,i)} \quad (10A),$$

$$\text{with } B_{r \text{ DIF}}(l,m)(k,i) = w^{((l m N/r + \tilde{N} (k/r^i) l r^i))_N} \quad (11A),$$

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1 = m = 0, ..., r - 1, i = 0, 1, ..., n - 1, k = 0, 1, ..., (N/r) - 1, ((x))_N denotes x modulo N and $\tilde{N}(k/r^i)$ is defined as the integer part of the division of k and r^i , whereby the operation of said butterfly processing element for said decimation in frequency radix-r butterfly is:

$$\text{the colon vector } X_{(r,k,i)} = B_{r \text{ DIF}} \times x = \begin{bmatrix} X_{(l,k,i)} \end{bmatrix} \quad (12A),$$

$$\text{where the } l^{\text{th}} \text{ output } X_{(l,k,i)} = \sum_{m=0}^{r-1} x_{(m)} w_{((l m N/r + \tilde{N}(k/r^i) l r^i))_N} \quad (13A).$$

11. An apparatus in accordance with claim 9, wherein said coefficients derived from the product of said adder matrix T_r and said twiddle factor matrix w_N^r , are substantially given by,

$$B_{r \text{ DIT}} = T_r \times W_{(r,k,i)} = \begin{bmatrix} B_{r \text{ DIT}}(l,m)_{(k,i)} \end{bmatrix} \quad (14A),$$

$$\text{in which } B_{r \text{ DIT}}(l,m)_{(k,i)} = w_{((l m N/r + \tilde{N}(k/r^{(n-i)}) m r^{(n-i)})_N)} \quad (15A),$$

$$\text{and } W_{(r,k,i)} = \begin{bmatrix} w_{(0,k,i)} & 0 & - & 0 \\ 0 & w_{(1,k,i)} & - & 0 \\ - & - & - & - \\ 0 & 0 & - & w_{((r-1),k,i)} \end{bmatrix} = \begin{bmatrix} w_{(l,m)_{(k,i)}} \end{bmatrix} \quad (16A),$$

where $w_{(l,m)_{(k,i)}} = w_{((\tilde{N}(k/r^{(n-i)}) m r^{(n-i)})_N)}$ for $l = m$, and 0 elsewhere (17A),

and $n = (\log N / \log r) - 1$, whereby the operation of said butterfly processing element for said decimation in time radix - r butterfly is:

$$\text{the colon vector } X_{(r,k,i)} = B_{r \text{ DIT}} \times x = \begin{bmatrix} X_{(l,k,i)} \end{bmatrix} \quad (18A),$$

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where the l^{th} output $X_{(l)}(k, i) = \sum_{m=0}^{r-1} x_{(m)} w^{((l m N/r + \tilde{N} (k/r^{(n-i)}))_m r^{(n-i)})_N}$ (19A).

12. A multistage Fourier transform processor responsive to N input data points to compute N output data points, said multistage Fourier transform processor comprising:

a first multiprocessor stage coupled to said N input data points, said first multiprocessor stage comprising a first plurality of Fourier transform processors, each of said first plurality of Fourier transform processors comprising

a plurality of r_1 parallel processing elements, each of said plurality of r_1 parallel processing elements having N/r_1 input data points and N/r_1 output data points, said N/r_1 input data points of said plurality of r_1 parallel processors being coupled to respective ones of said N input data points, each of said plurality of r_1 parallel processing elements being independent of each other and having at least two stages of butterfly computing elements; and

a combination phase computing element having N output points, said combination phase computing element having N input data points coupled to respective ones of said N/r_1 output data points from said plurality of r_1 processing elements, said combination phase computing element including a single stage of butterfly computing elements; and

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a/ a second multiprocessor stage coupled to said N output data points of said first multiprocessor stage, said second multiprocessor stage comprising a second plurality of Fourier transform processors, each of said second plurality of Fourier transform processors comprising,

a plurality of r_2 parallel processing elements, each of said plurality of r_2 parallel processing elements having N/r_2 input data points and N/r_2 output data points, said N/r_2 input data points of said plurality of r_2 parallel processors being coupled to respective ones of said N output data points of said first multiprocessor stage, each of said plurality of r_2 parallel processing elements being independent of each other and having at least two stages of butterfly computing elements; and

a combination phase computing element having N output points forming said N data output data points of said Fourier transform processor, said combination phase computing element having N input data points coupled to respective ones of said N/r_2 output data points from said plurality of r_2 processing elements, said combination phase computing element including a single stage of butterfly computing elements.

13. A multistage Fourier transform processor in accordance with claim 12, further including a plurality of (i) multiprocessor stages, wherein the radix r FFT of each said plurality of (i) multiprocessor stages is expressed in terms of its i^{th} partial FFTs as follows:

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$$X_{(k)} = T_r \hat{*} \left[w_N^{j_1 k} T_r \hat{*} \left[w_{N/r}^{j_2 k} \dots T_r \hat{*} \left[w_{N/r^{(i-1)}}^{j_i k} \left[\sum_{n=0}^{(N/r^i)-1} X_{(r^i n + r^{(i-1)} j_i + \dots + r^0 j_1)} w_{N/r^i}^{nk} \right] \right] \right] \right] \dots \quad (43),$$

for $j_1 = \dots = j_i = 0, 1, \dots, r-1$ and $i = 1, \dots, p$.

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